

惑星科学的衝突過程における

物質強度・空隙モデル  
と

数値シミュレーションと実験の関わり

神戸大・理

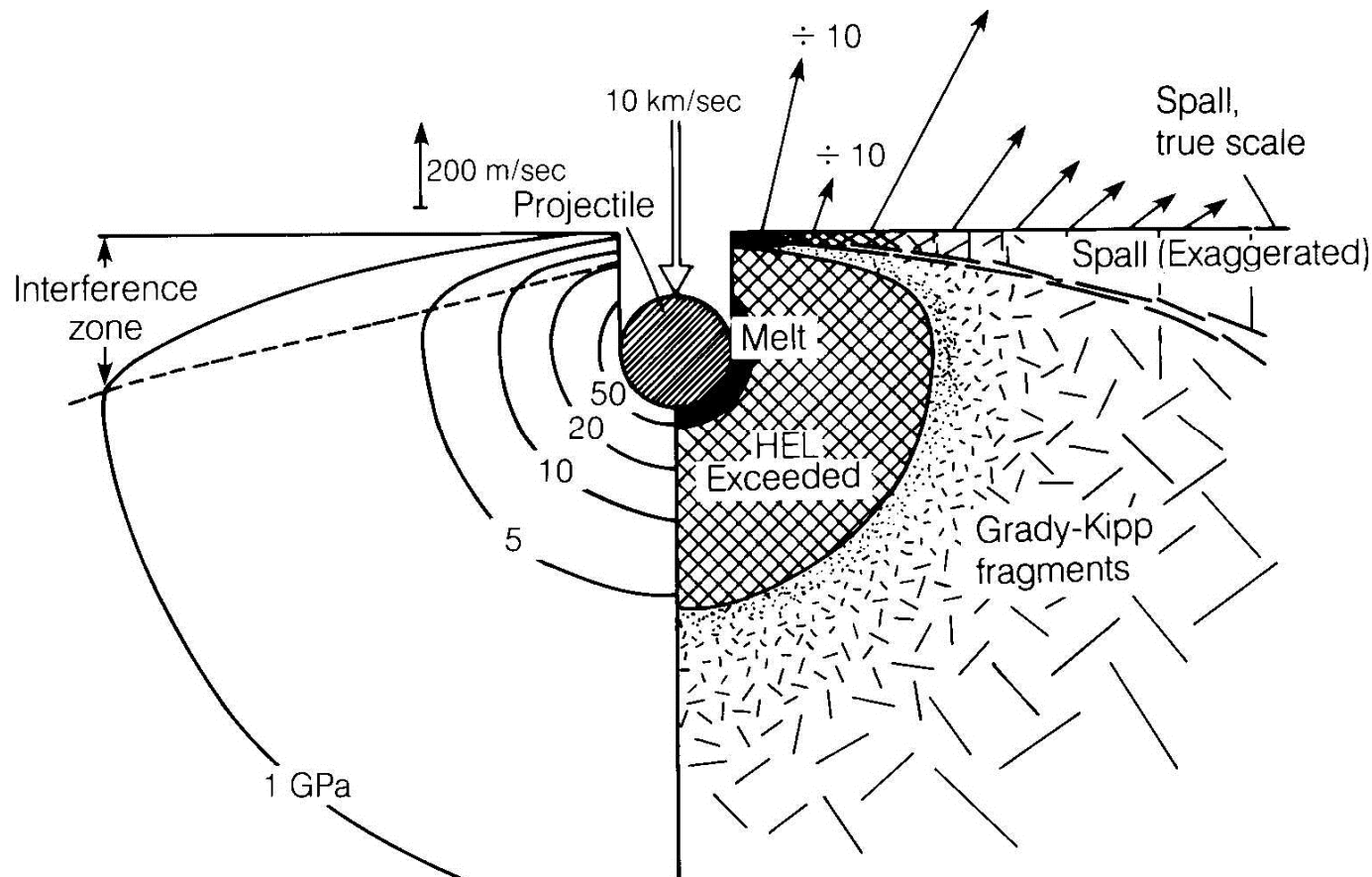
中村昭子

# 内容

1. 強度・空隙モデルが適用される問題
2. 強度モデル、空隙モデル
3. 数値シミュレーションと実験の関わり

# 1. 強度・空隙モデルが適用される問題

$$P \leq \rho C^2$$



(Melosh, 1989)

# 小惑星帯の衝突進化とクレーター年代

O'Brien, Greenberg, and Richardson, 2006

- **Gaspra** (S, 19x12x11 km)
- **Ida** (S, 56x24x21 km)
- **Mathilde** (C, 66x48x44 km) > 4Gyr
- **Eros** (S, 34x11x11 km)

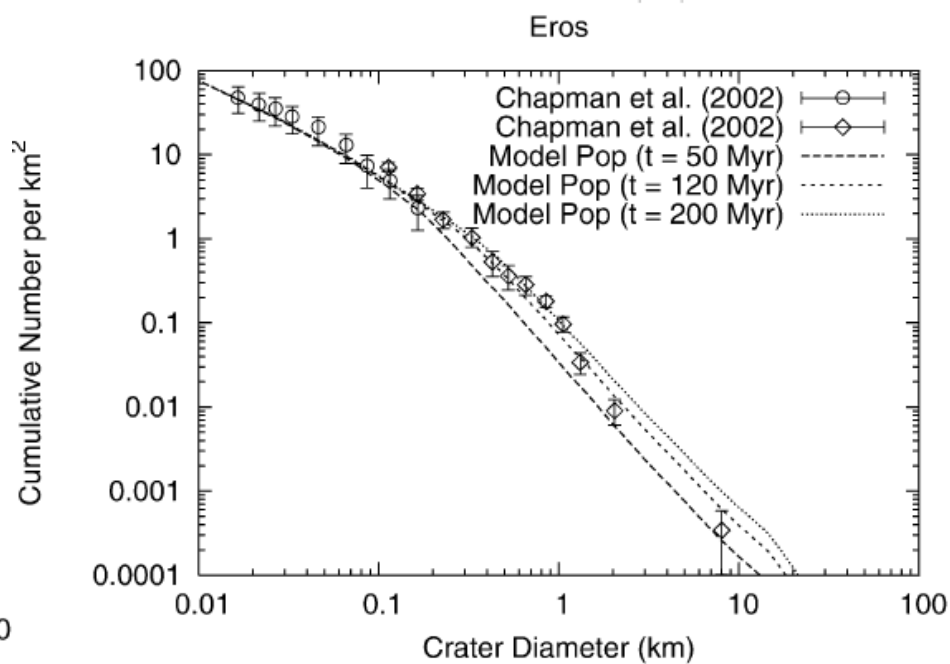
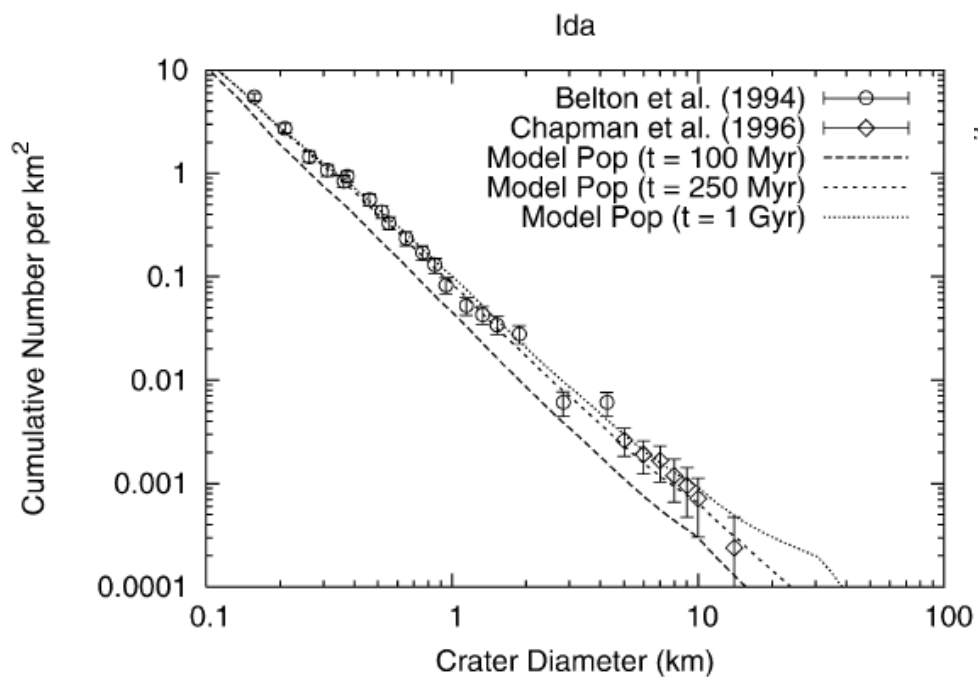
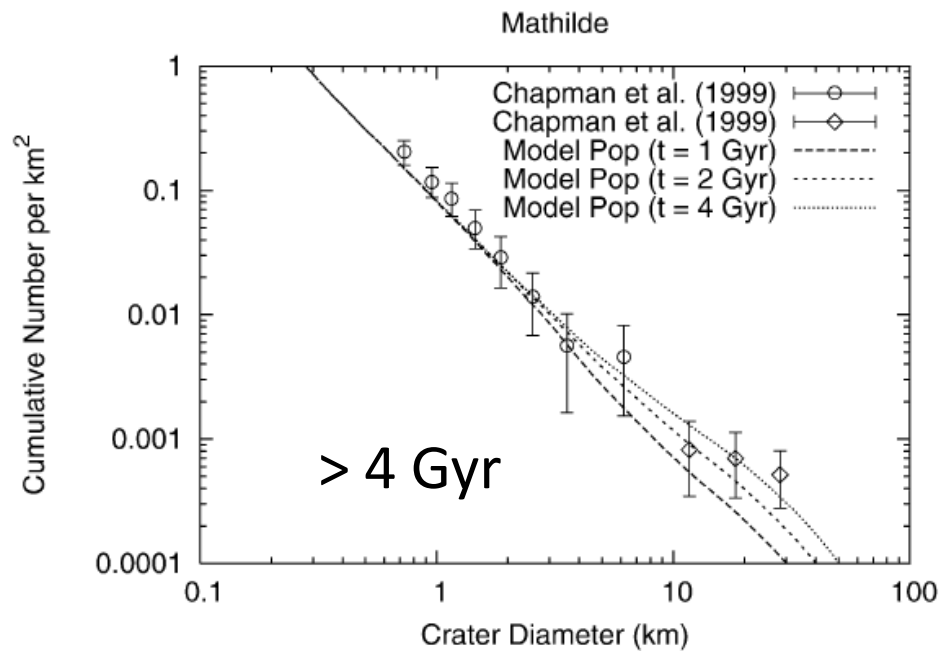
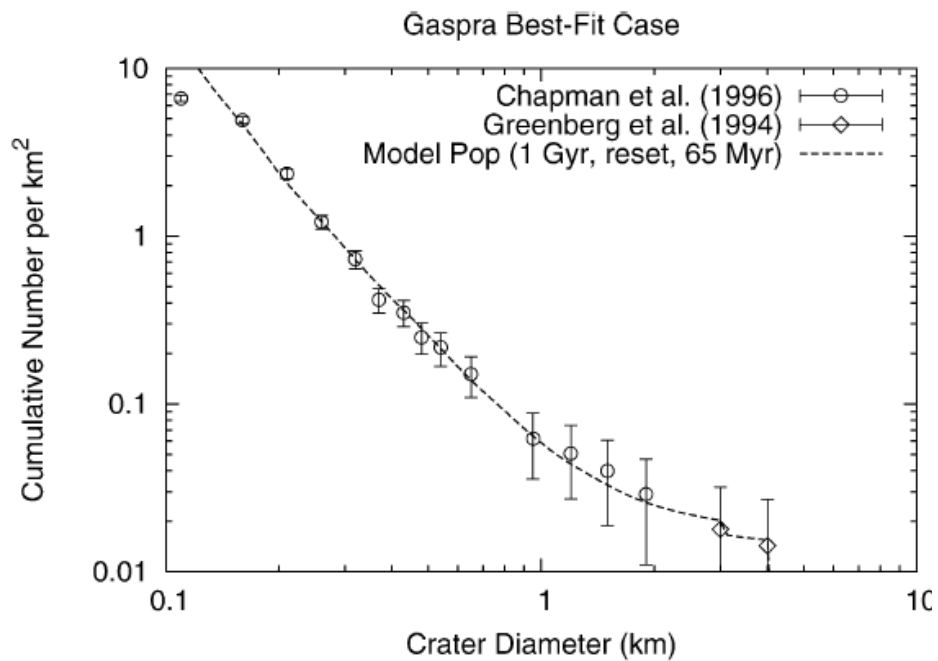


Michel, O'Brien, and Hirata, 2008

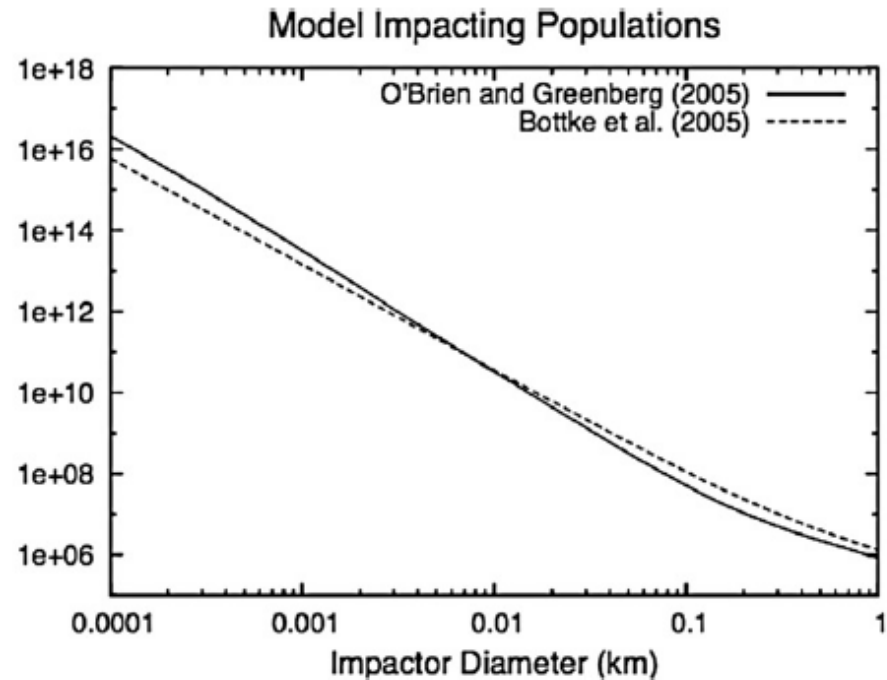
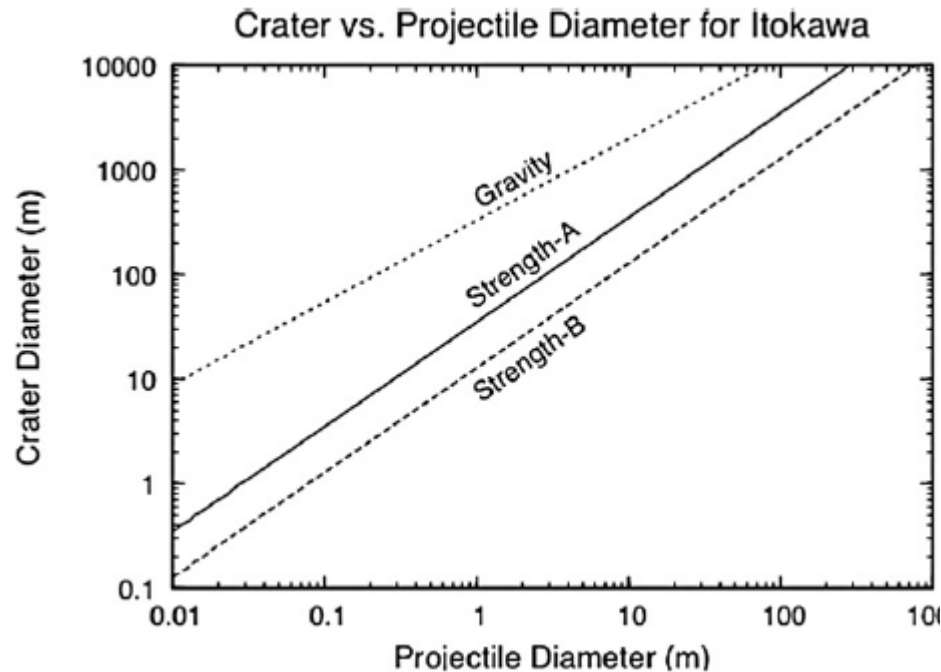
**Itokawa** (S, 0.45x0.29x0.21 km) 200 Myr <, < 2 Gyr

**Eros**

Lutetia, Vesta, ...



(Michel, O'Brien, Hirata, 2008)



**strength-A:** hydrocode (SALE)

(Nolan, Asphaug, Melosh, and Greenberg, 1996)

**gravity:** pi-group hardrock (Holsapple, 1993)

**strength-B:** pi-group hardrock (Holsapple, 1993.)

O'Brien et al. (2006)

Mathilde に対して

(仕方なく) pi-group loose sand (Holsapple 1993)

を適用

### 小惑星帯の衝突進化モデル

O'Brien et al., 2005; Bottke et al., 2005

制約:

小惑星サイズ分布

・ベスタが壊れていないが大きなクレーター

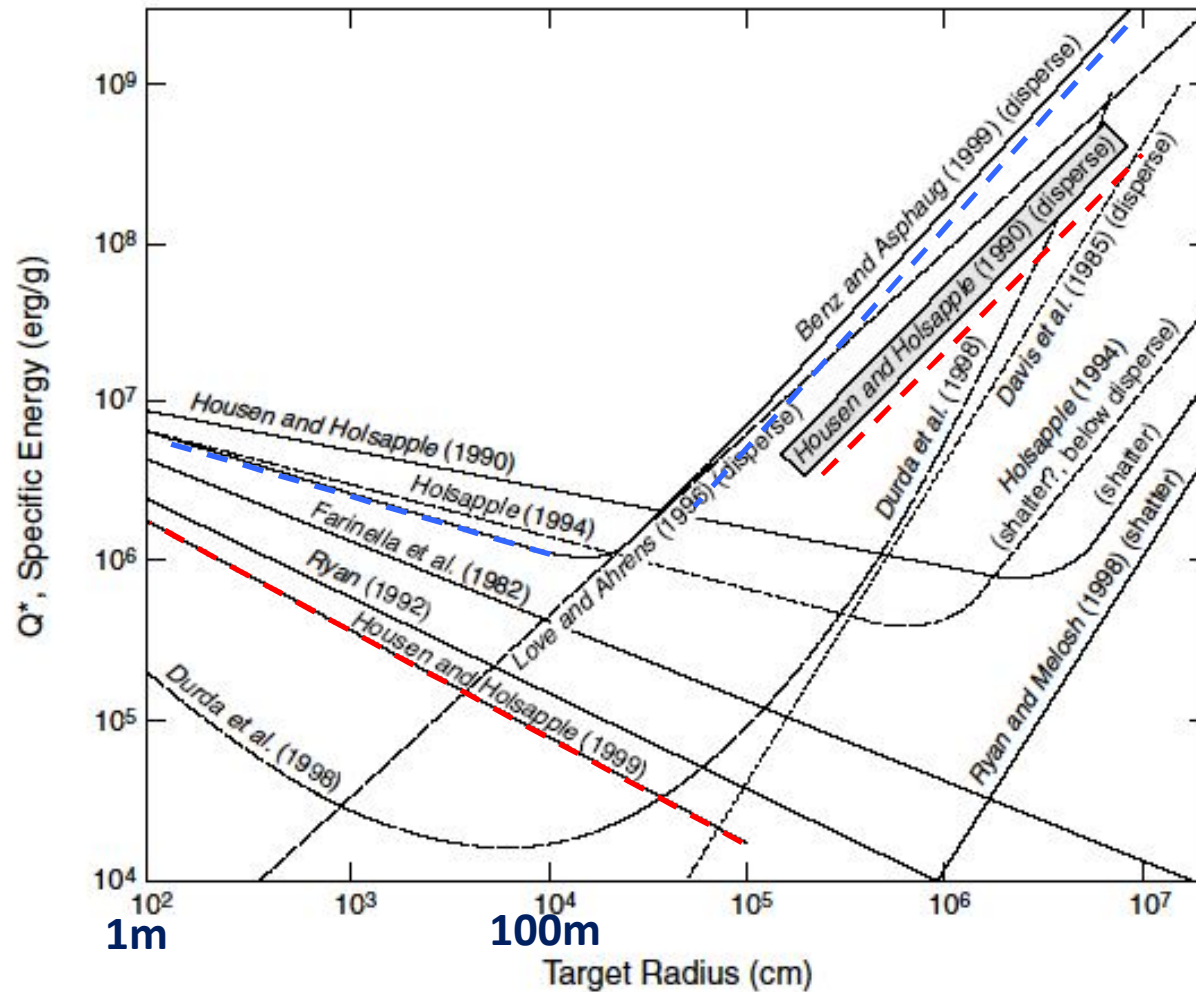
・普通コンドライトの宇宙線照射年代

・小惑星 (Gaspra, Ida, Mathilde, Eros) クレーター

サイズ頻度分布

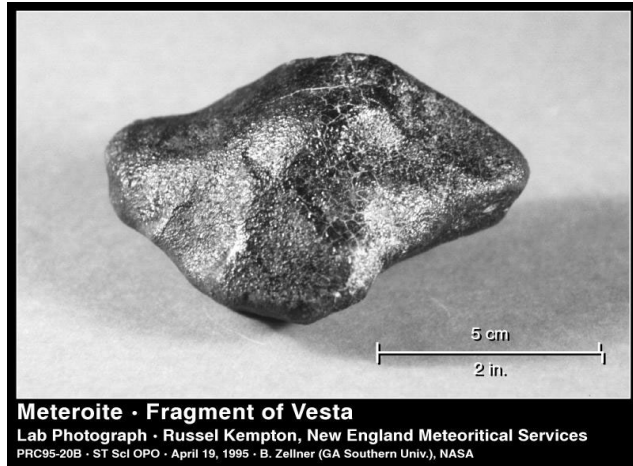
・小惑星の族の個数

# スケーリング則や数値シミュレーションにより 推定された衝突破壊のしきい値 ( $Q^*$ )



(Holsapple et al., 2002)

## 2. 強度モデル、空隙モデル



流体

連続体

粉粒体

References: Holsapple, K. A., 2009.

On the “strength” of the small bodies of the solar system:  
A review of strength theories and their implementation for analyses  
of impact disruptions.

+ iSALE manual + ,,,



# 岩石強度・空隙モデルの入った 数値シミュレーション

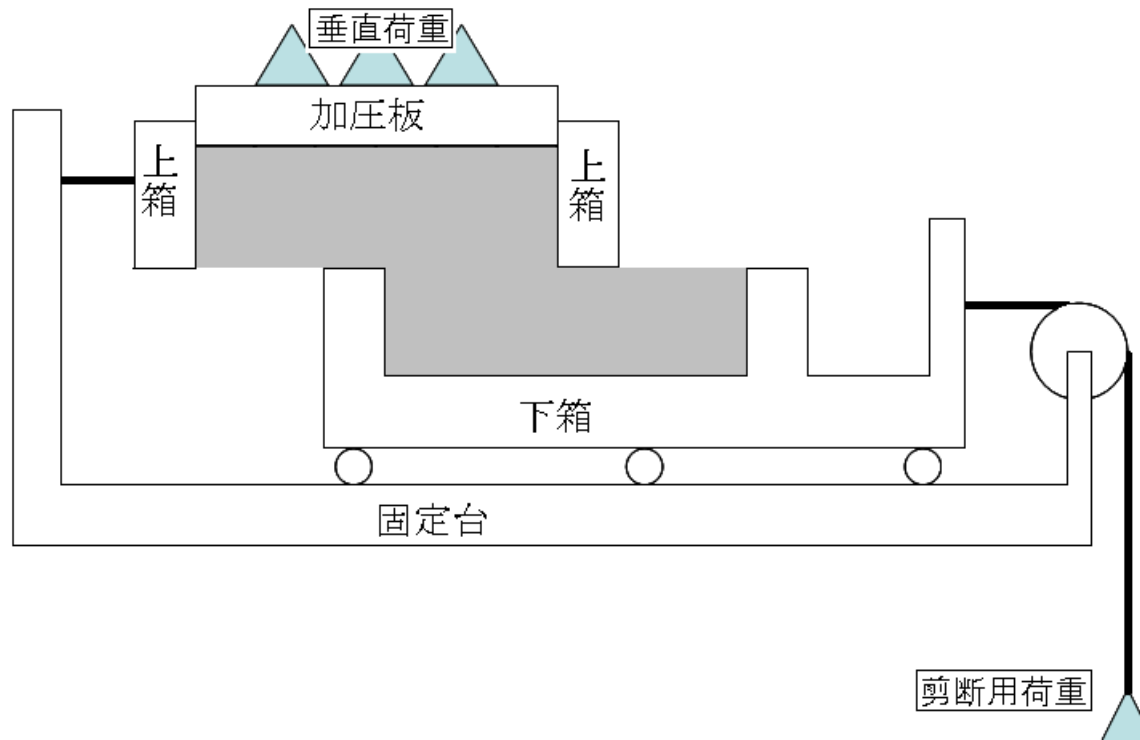
- Dynamic fragmentation in impacts - Hydrocode simulation of laboratory impacts, Melosh, Ryan, and Asphaug, 1992. **SALE + G-K モデル**
- Impact simulations with fracture.I-Method and tests, Benz and Asphaug, 1994. **G-K モデル + SPH**
  - > Catastrophic disruptions revisited, Benz and Asphaug, 1999. **Q\***  
(解像度に難あり、後にBenz自身がCD6(2003)かCD7(2007)で言及)
- Modeling damage and deformation in impact simulations, Collins, Melosh, and Ivanov, 2004. **SALE + Collins damage model**

*“more comprehensive model (than the model in ‘92)” by Holsapple (2009)*

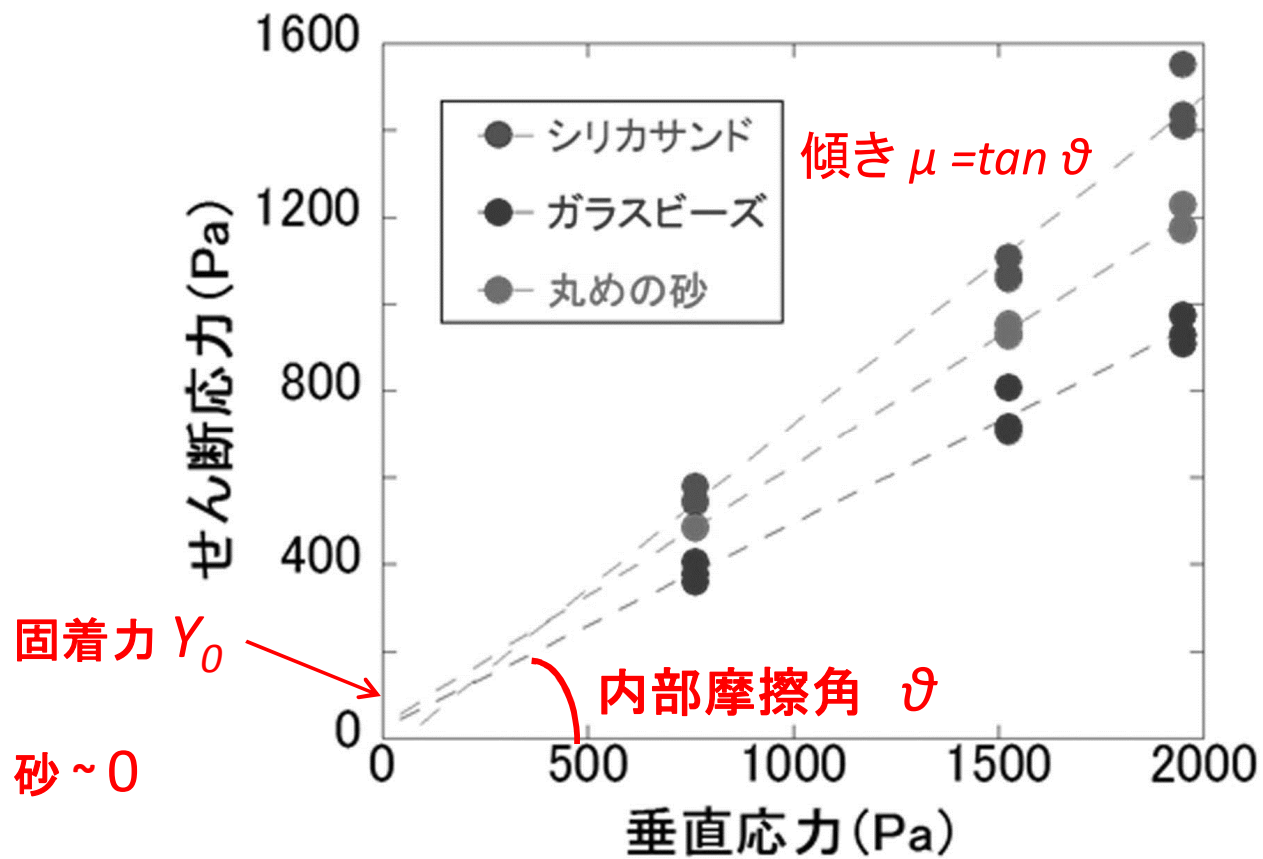
- A strain-based porosity model for use in hydrocode simulations of impacts and implications for transient crater growth in porous targets, Wunnenmann, Collins, and Melosh, 2006.  **$\epsilon$ - $\alpha$  model**
- Numerical simulations of impacts involving porous bodies, Jutzi, Benz, and Michel, 2008. **P- $\alpha$  model**

# 粉粒体のせん断強度

## 剪断強度測定器 概略図

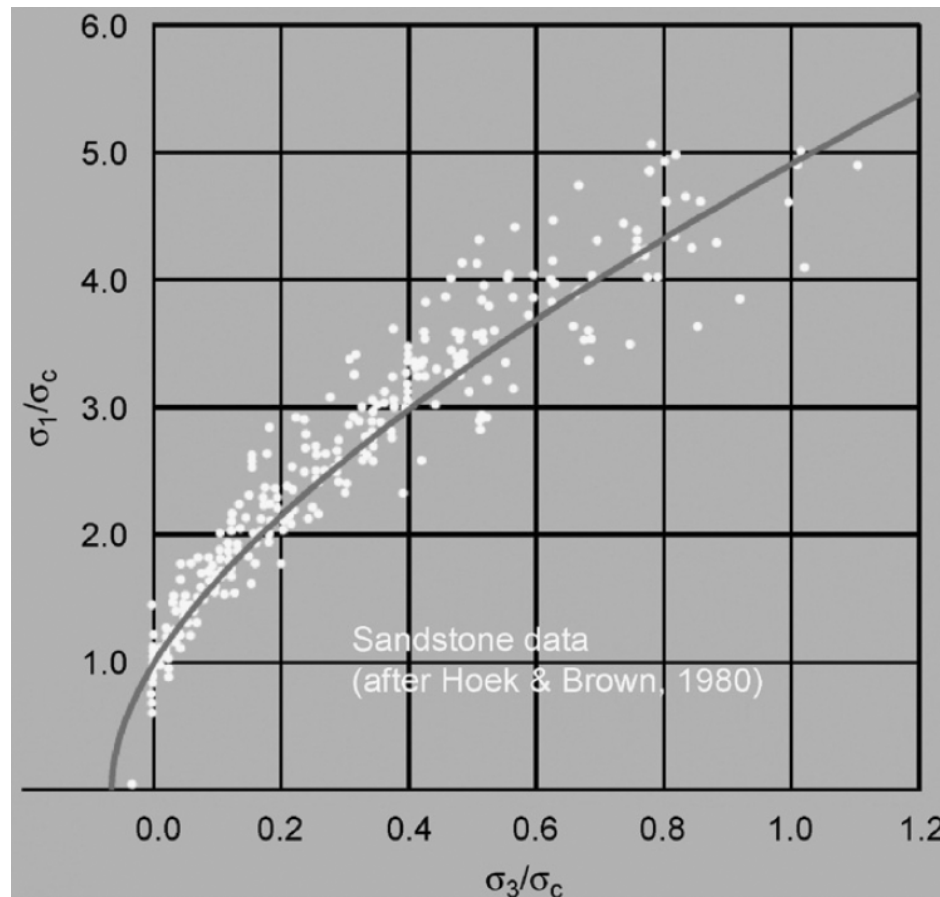


描画: Hakura, 2011による

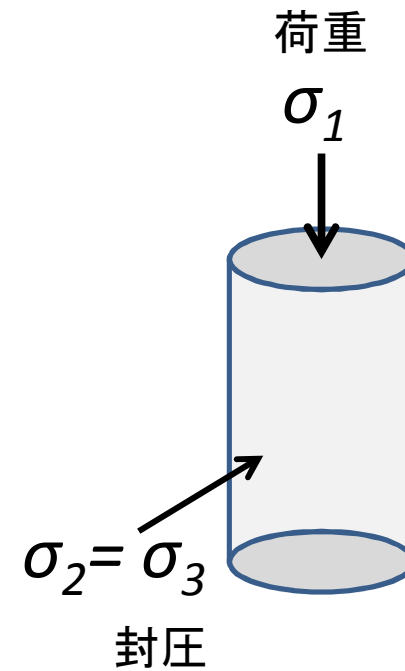


(Aoki, 2014)

# 砂岩の三軸圧縮試験結果



(Holsapple, 2009)



$\sigma_1$ : 最大主応力

$\sigma_3$ : 最小主応力

# 強度

それを超えると、変形が元に戻らない応力

$$f(\sigma) = C.$$

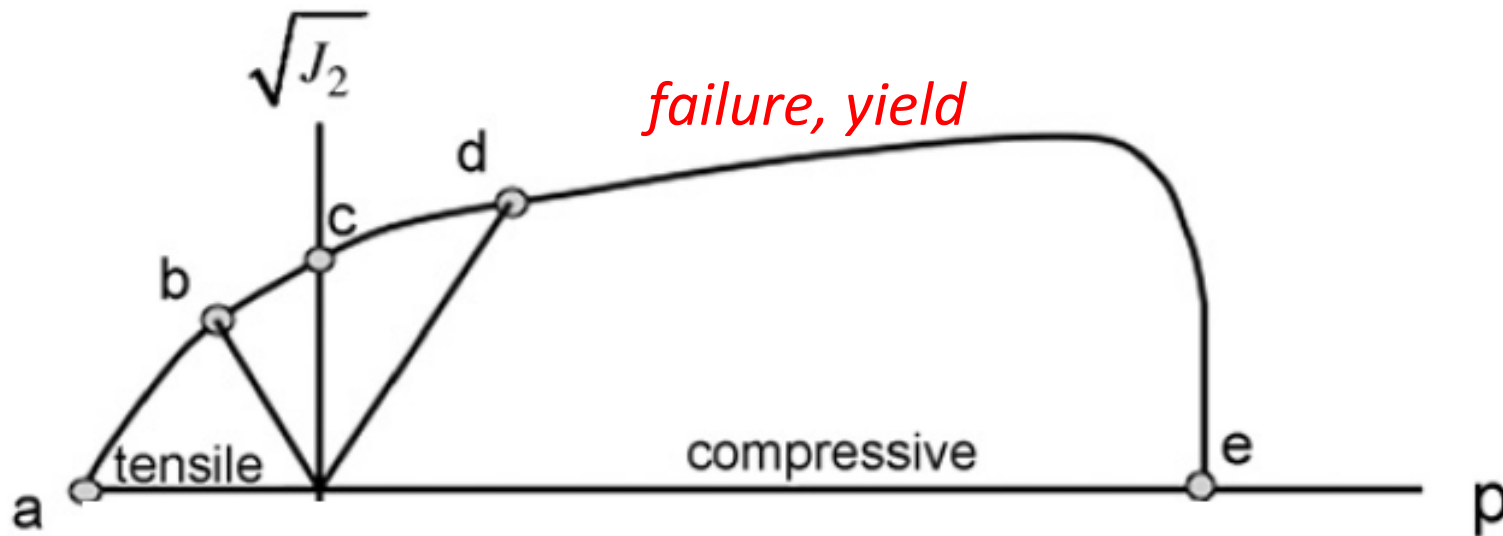
応力の不変量  $J_1, J_2, J_3$  を用いて、

$$f(J_1, J_2, J_3) = f(P, J_2) = C.$$

$$J_1 = \sigma_1 + \sigma_2 + \sigma_3 = -3P$$

$$J_2 = \frac{1}{6} ((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)$$

$$f(P, J_2) = C \longrightarrow \boxed{\sqrt{J_2} = Y} \quad \begin{matrix} \text{iSALE} \\ Y: \text{強度モデル} \end{matrix}$$



- b: 一軸引張強度 ( $\sigma_1 = \sigma, \sigma_2 = \sigma_3 = 0$ )  
 $J_2 = \sigma/\sqrt{3}, \quad P = -\sigma/3$
- c: 固着力
- d: 一軸圧縮強度

# iSALEの物質パラメタ (material.inp)

MATNAME Material name : mygrani  
EOSNAME EOS name : granite  
EOSTYPE EOS type : aneos

**STRMOD Strength model : ROCK**  
**DAMMOD Damage model : COLLINS**  
ACFL Acoustic fluidisation : BLOCK  
**PORMOD Porosity model : NONE**  
THSOFT Thermal softening : OHNAKA  
LDWEAK Low density weakening : POLY

**強度、ダメージ、空隙率**

-----  
POIS pois : 3.D-01  
TMELT0 tmelt0 : 1.673D+03  
CHEAT C\_heat : 1.D+03  
TFRAC tfrac : 1.2D+00  
ASIMON a\_simon : 6.D+09  
CSIMON c\_simon : 3.D+00  
YDAM0 ydam0 (ycoh) : 1.D+04  
FRICDAM fricdam : 8.D-01  
YLIMDAM ylimdam : 2.D+09  
**YINT0 yint0 : 1.D+07**  
**FRICINT fricint : 1.1D+00**  
**YLIMINT ylimint : 2.5D+09**  
BDTPRES bdt\_pres : -1.D+00  
BPTPRES bpt\_pres : -1.D+00  
GAMETA gam\_eta : 8.D-03  
GAMBETA gam\_beta : 1.15D+02

## 強度モデル (STRMOD)

**ROCK** Pressure- and damage-dependent strength model for rock-like materials.

**DRPR** Drucker-Prager: Linear pressure-dependent strength model for granular materials.

**LUNDI** Lundborg intact: Non-linear pressure-dependent strength model for intact rock.

**LUNDD** Lundborg damaged: Non-linear pressure-dependent strength model for damaged rock.

**VNMS** Von Mises: Constant yield-strength model for ductile materials.

**JNCK** Johnson and Cook: Strain and strain-rate dependent strength model for metals.

**LIQU** Liquid: Newtonian fluid model

**HYDRO** Hydrodynamic: Inviscid fluid model



**DRPR** Drucker-Prager:  
Linear pressure-dependent strength model  
for granular materials.

$$Y = \min(Y_0 + \mu p, Y_m)$$

Constant	Input parameter	Description
$Y_0$	YDAM0	Cohesion (yield strength at zero pressure)
$\mu$	FRICDAM	Coefficient of internal friction for material
$Y_m$	YLIMDAM	Limiting strength at high pressure

**LUIND** Lundborg intact:  
Non-linear pressure-dependent strength  
model for intact rock.

$$Y = Y_0 + \frac{\mu p}{1 + \frac{\mu p}{Y_m - Y_0}}$$

Constant	Input parameter	Description
$Y_0$	YINT0	Cohesion (yield strength at zero pressure)
$\mu$	FRICINT	Coefficient of internal friction for material
$Y_m$	YLIMINT	Limiting strength at high pressure

**ROCK** Pressure- and damage-dependent strength model for rock-like materials.

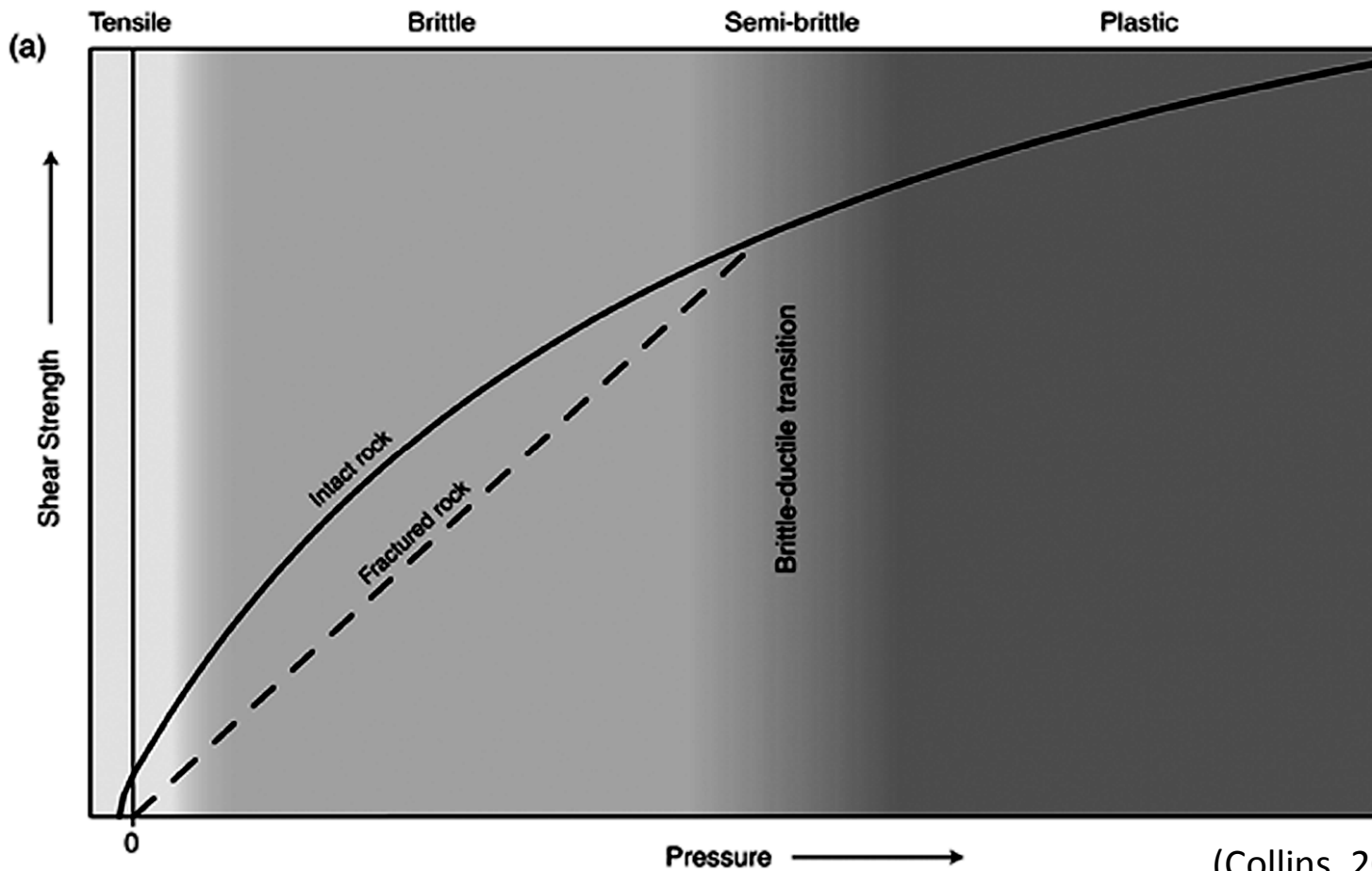
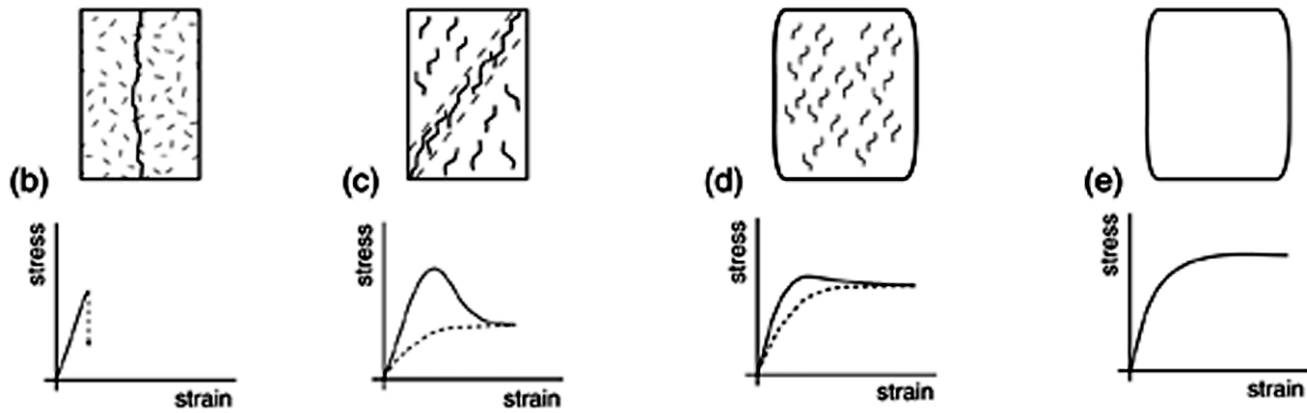
$$Y = Y_d D + Y_i(1. - D)$$

*D*: damage parameter, *D*=0: intact, *D*=1: fully-damaged

$$Y_d = \min(Y_{d0} + \mu_d p, Y_{dm})$$

$$Y_i = Y_{i0} + \frac{\mu_i p}{1 + \frac{\mu_i p}{Y_{im} - Y_{i0}}}$$

Constant	Input parameter	Description
$Y_{i0}$	YINT0	Cohesion of intact material
$\mu_i$	FRICINT	Coefficient of internal friction for intact material
$Y_{im}$	YLIMINT	Limiting strength at high pressure for intact material
$Y_{d0}$	YDAM0	Cohesion of damaged material
$\mu_d$	FRICDAM	Coefficient of internal friction for damaged material
$Y_{dm}$	YLIMDAM	Limiting strength at high pressure for damaged material



(Collins, 2004)

## ダメージモデル(DAMMOD)

**COLLINS** Combined shear and tensile failure model with brittle, semi-brittle and ductile shear failure regimes.

**IVANOV** Shear failure model with pressure-dependent failure strain.

**SIMPLE** Shear failure model with constant failure strain.

**NONE** No damage model; material remains intact.

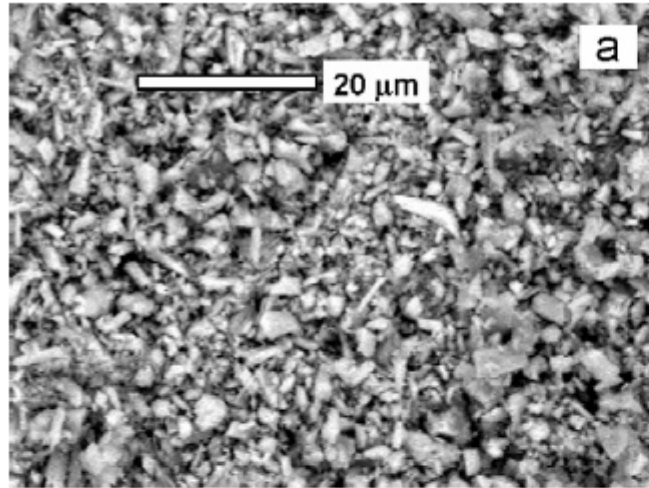
**SIMPLE** Shear failure model with constant failure strain.

$$D = \min \left( \frac{\epsilon_p}{\epsilon_f}, 1 \right)$$

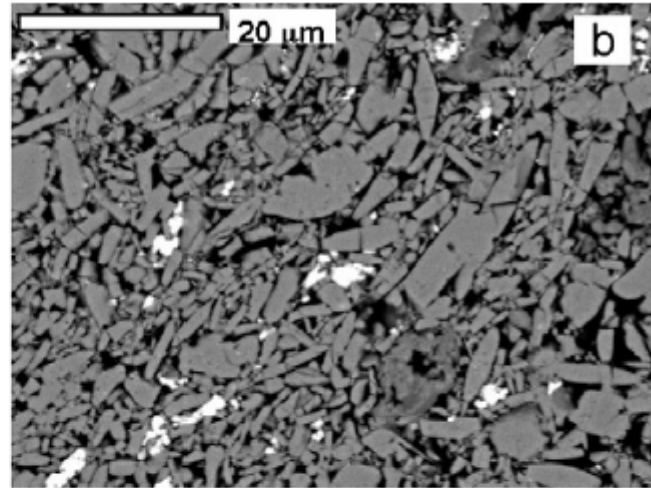
$\epsilon_p$  TPS - Total plastic strain (shear)

Constant	Input parameter	Description
$\epsilon_f$	FAILSTRN	Constant plastic strain at failure

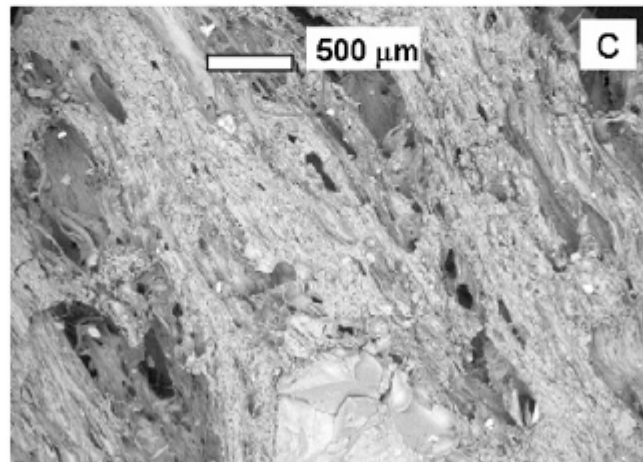
# 空隙モデル



石膏



Allende matrix



軽石

(Nakamura et al., 2009)

“distension (膨張パラメタ)”  $\alpha$  を使って  
1つの連続体で表す

$\alpha$  = 物質の真密度  $\rho_s$  / バルク密度  $\rho$

$$P = f(\rho, E, \alpha) = \frac{1}{\alpha} P_s(\alpha\rho, E) = \frac{1}{\alpha} P_s(\rho_s, E)$$

熱力学的説明 see Holsapple (2008)

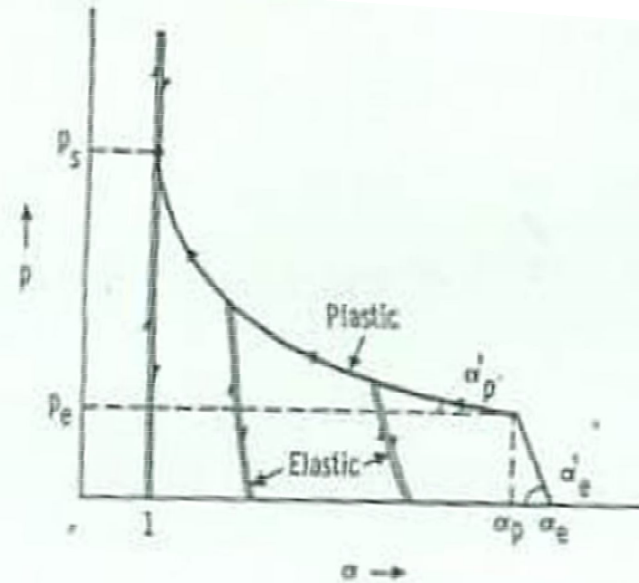
$P$ - $\alpha$  モデル (Hermann, 1969)

$\varepsilon$ - $\alpha$  モデル (Wunnemann et al., 2006)



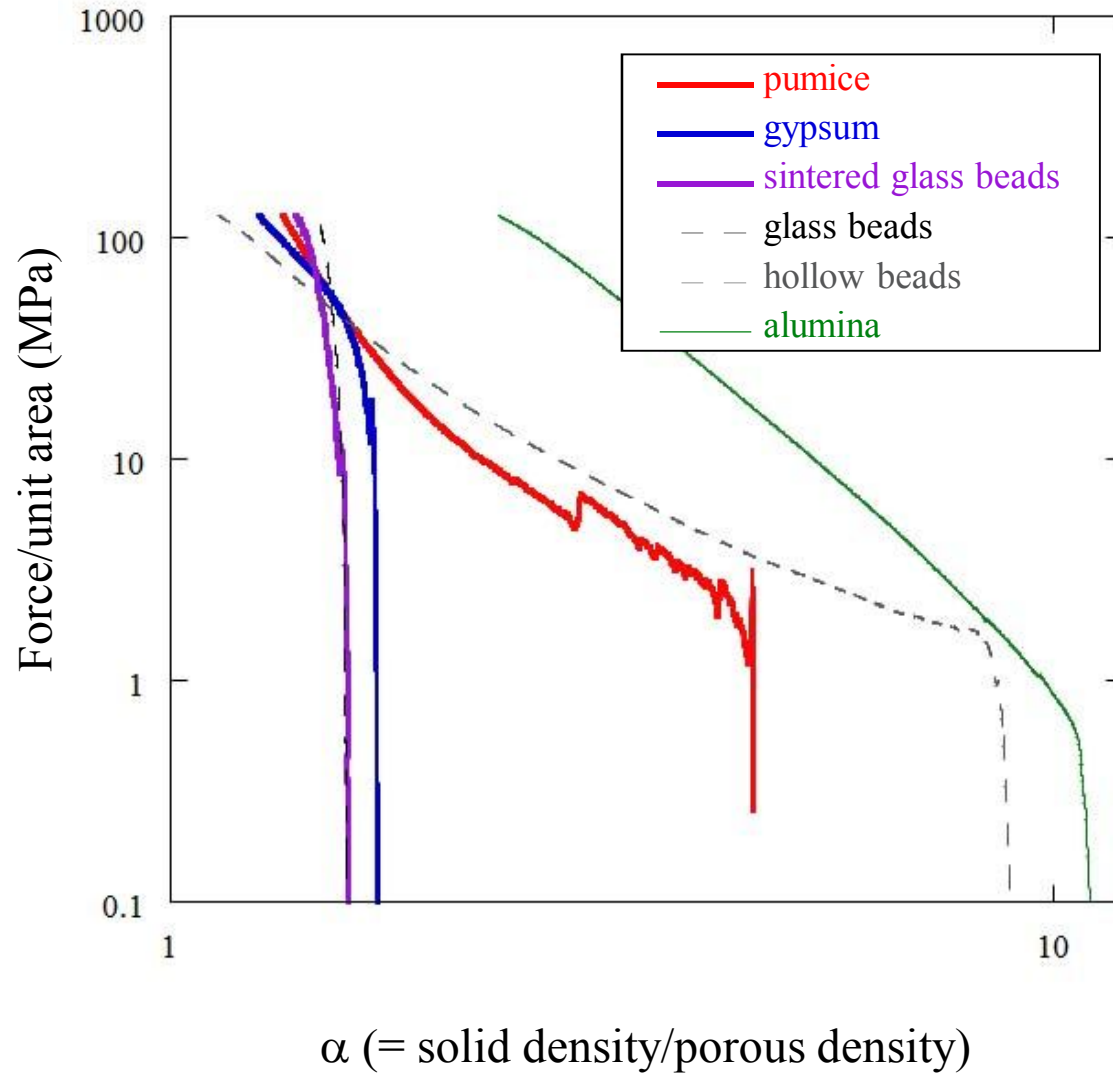
# $P$ - $\alpha$ モデル (Hermann, 1969)

FIG. 1. Schematic of the postulated compaction behavior of a ductile porous material.



$$\alpha = \rho_s / \rho = F(P)$$

# 多孔質物質の圧密曲線

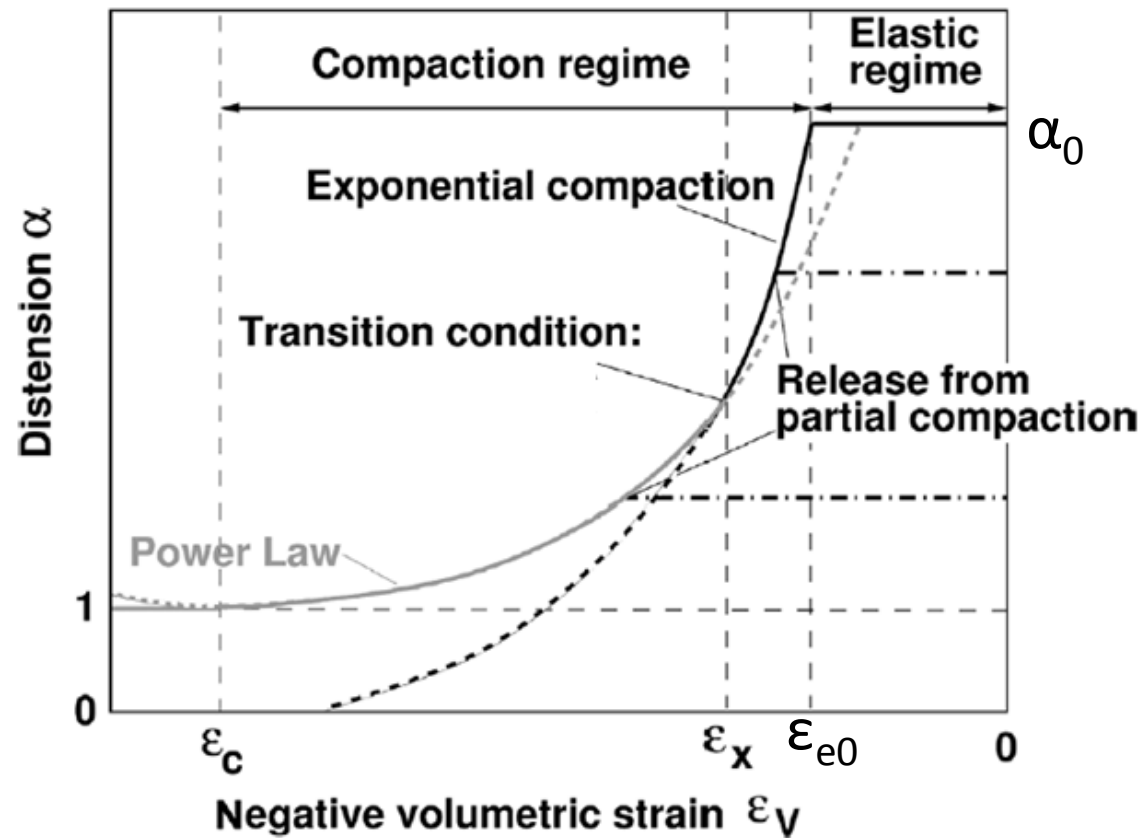


head speed=0.06mm/min

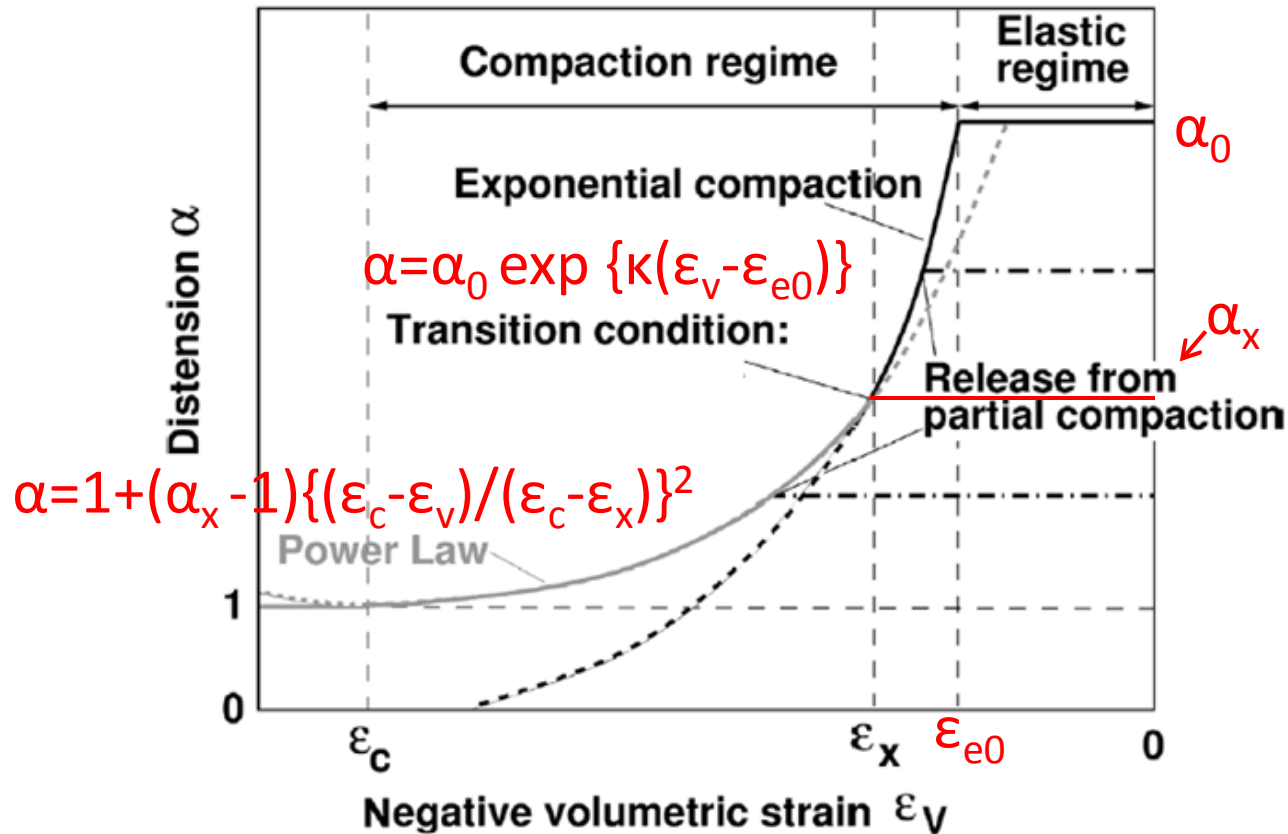
Hiraoka and Nakamura, 2007  
(@CD8)

# $\varepsilon$ - $\alpha$ モデル

$$\varepsilon_V = \int_{V_0}^{V'} \frac{dV}{V} = \ln\left(\frac{V'}{V_0}\right) = \ln\left(\frac{V'}{V_S} \frac{V_S}{V_0}\right) = \ln\left(\frac{\alpha}{\alpha_0}\right),$$



( Wünnemann et al, 2006)



Note that the behaviour of the original model can be regained by choosing  $\chi = 1$ .

Constant	Input parameter	Description
$\alpha_0$	ALPHA0	Initial distension of porous material ( $1/(1-\text{porosity})$ )
$\epsilon_{e0}$	EPSE0	Elastic volumetric strain threshold (-ve in compression)
$\alpha_x$	ALPHAX	Distension at transtion from exponential to power-law compaction
$\kappa$	KAPPA	Compaction rate parameter in exponential compaction regime
$\chi$	CHI	Ratio of porous to solid material sound speed at zero pressure

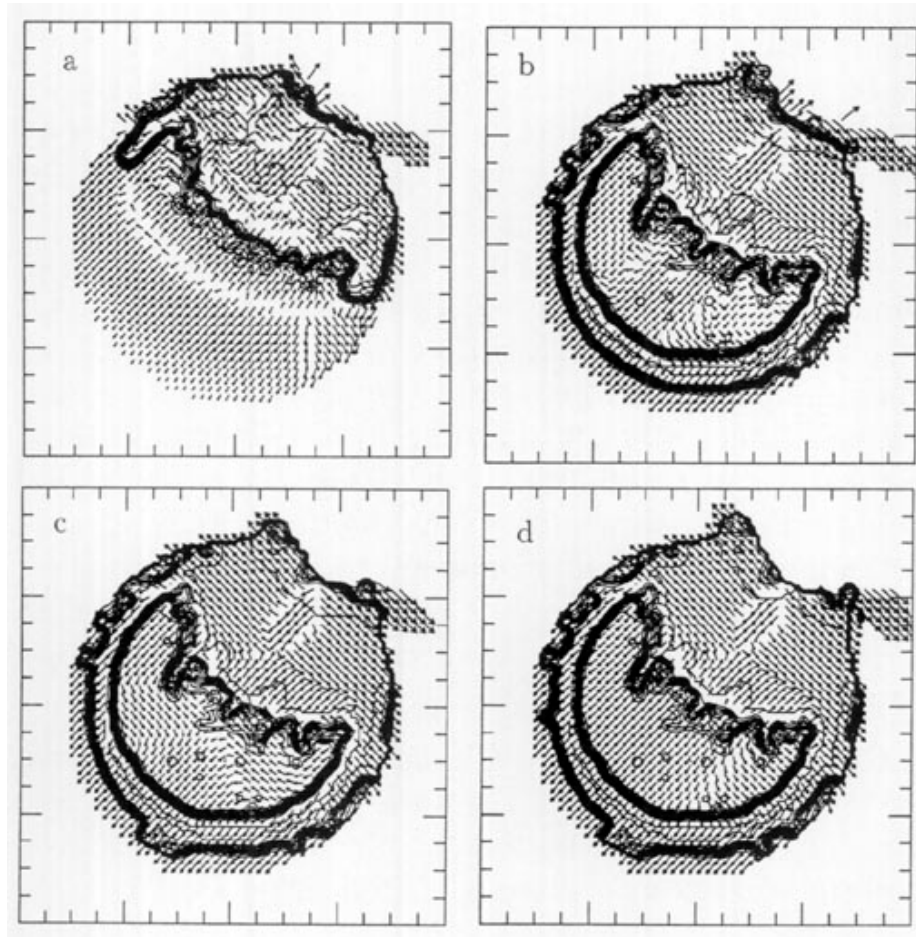
### 3. 数値シミュレーションと実験の関わり

*Dynamic fragmentation in impacts - Hydrocode simulation of laboratory impacts, Melosh, [Ryan](#), and Asphaug, 1992. SALE + G-K*

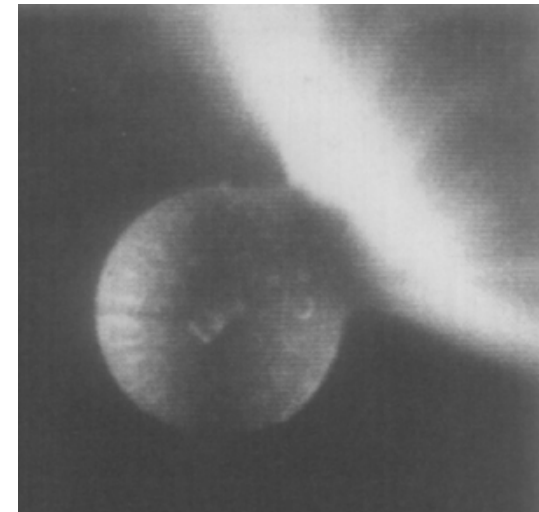
- Impact simulations with fracture.I-Method and tests, Benz and Asphaug, 1994. *G-K + SPH*  
ref. Nakamura and Fujiwara, 1991; Nakamura 1993
- Weibull parameters of Yakuno basalt targets used in documented high-velocity impact experiments, Nakamura, Michel, and Setoh, 2007. G-KのWeibull 定数
- Numerical simulations of impacts involving porous bodies, Jutzi, Benz, and Michel, 2008. *P- $\alpha$  model*

Numerical simulations of impacts involving porous bodies II. Comparison with laboratory experiments, Jutzi, Michel, Hiraoka, Nakamura, Benz, 2009.

# 玄武岩へのナイロン球衝突

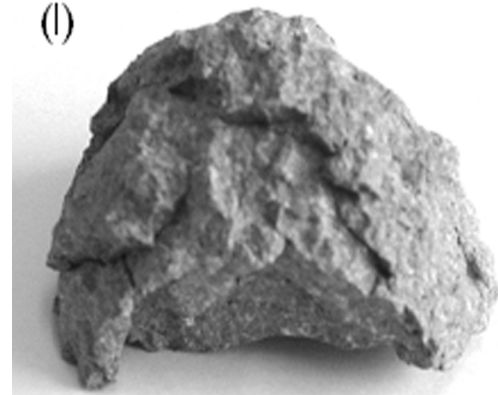


衝突から10、20、30、40  $\mu\text{s}$ の等ダメージ線と  
粒子速度ベクトル (Benz and Aphaug, 1994)



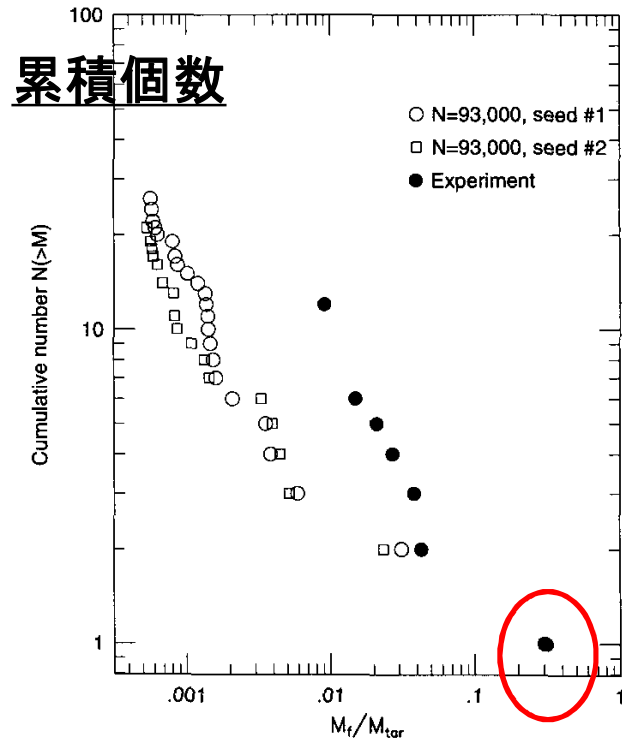
衝突から100  $\mu\text{s}$  (Nakamura and  
Fujiwara, 1991; Nakamura, 1993)

(I)

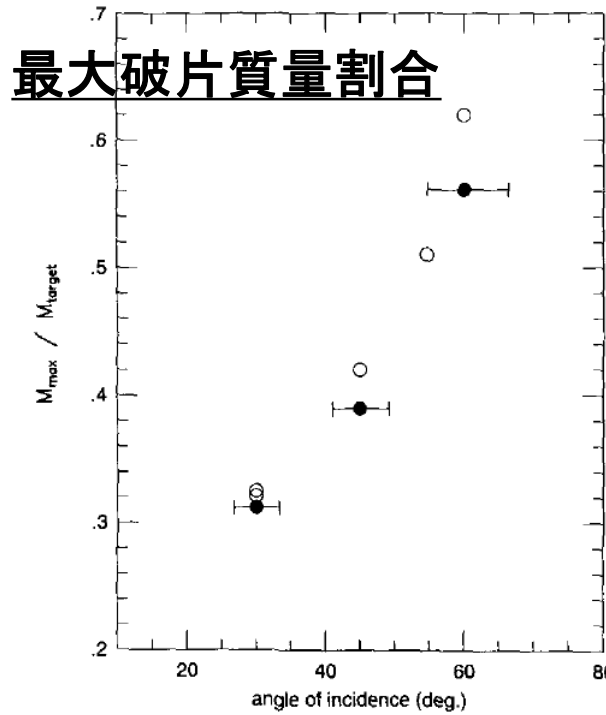


2cm

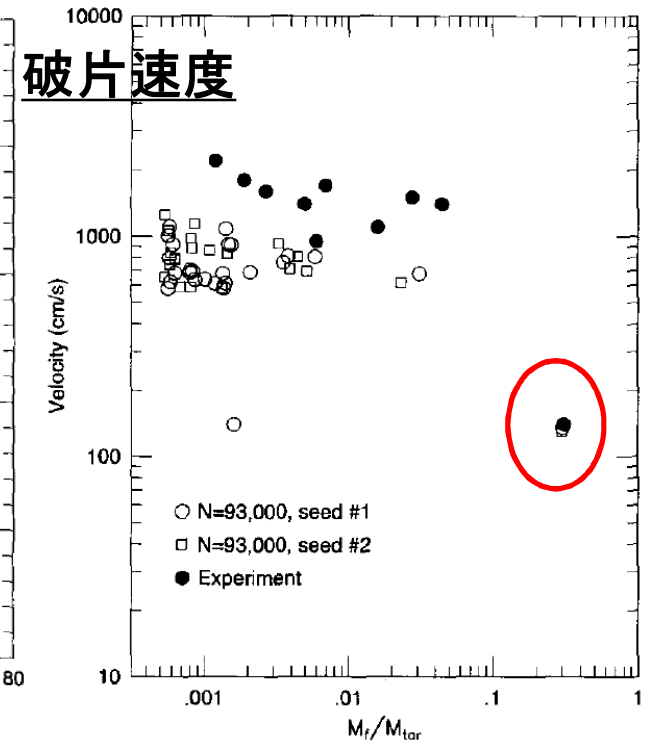
# Benz and Asphaug 1994 数值シミュレーションと 実験との比較



破片質量/標的質量



衝突角度



破片質量/標的質量

# Weibullパラメタ (m,k) の値

TABLE 1. Weibull dynamic fracture coefficients for various rocks.

Material	Reference	m	k (cm <sup>-3</sup> )	ln(k)/m
Basalt*	<i>Melosh et al. (1992)</i>	9.5	$1.0 \times 10^{27}$	6.54
Basalt*	<i>Benz and Asphaug (1995)</i>	9.0	$4.0 \times 10^{29}$	7.17
Basalt†	<i>Lindholm et al. (1974)</i>	9.5	$1.59 \times 10^{30}$	7.32
Granite†	<i>Grady and Lipkin (1980)</i>	6.2	$4.14 \times 10^{17}$	6.54
Water Ice*,†,‡	<i>Benz and Asphaug (1999)</i>	9.6	$1.4 \times 10^{32}$	7.71
30% Sand + Water Ice†	<i>Stewart et al. (1999)</i>	9.57	$1.34 \times 10^{30}$	7.25
Concrete†	<i>Grady and Lipkin (1980)</i>	5.3	$5.27 \times 10^{12}$	5.53
Oil Shale†	<i>Grady and Kipp (1980)</i>	8.1	$1.70 \times 10^{21}$	6.04
Limestone†	<i>Grady and Lipkin (1980)</i>	57.0	$4.26 \times 10^{167}$	6.77

\* Determined from simulation fits to laboratory data. The two-dimensional axisymmetric simulations of *Melosh et al. (1992)* require stronger fracture parameters than the nonsymmetric three-dimensional simulations of *Benz and Asphaug (1995)* for the same impact experiment.

† Determined experimentally through measurements of tensile strength vs. strain rate.

‡ Earlier published values of  $m = 8.7$ ,  $k = 3.2 \times 10^{38}$  (*Lange and Ahrens, 1983*) were later corrected to similar values [ $m = 9.57$ ,  $k = 1.28 \times 10^{32}$  (*Stewart et al., 1999*)].

(Asphaug et al., 2002)



# Weibull (1939)分布

応力 $\sigma$ 以下で成長し始めるひびの数密度を

$$n(\sigma) = k \sigma^m$$

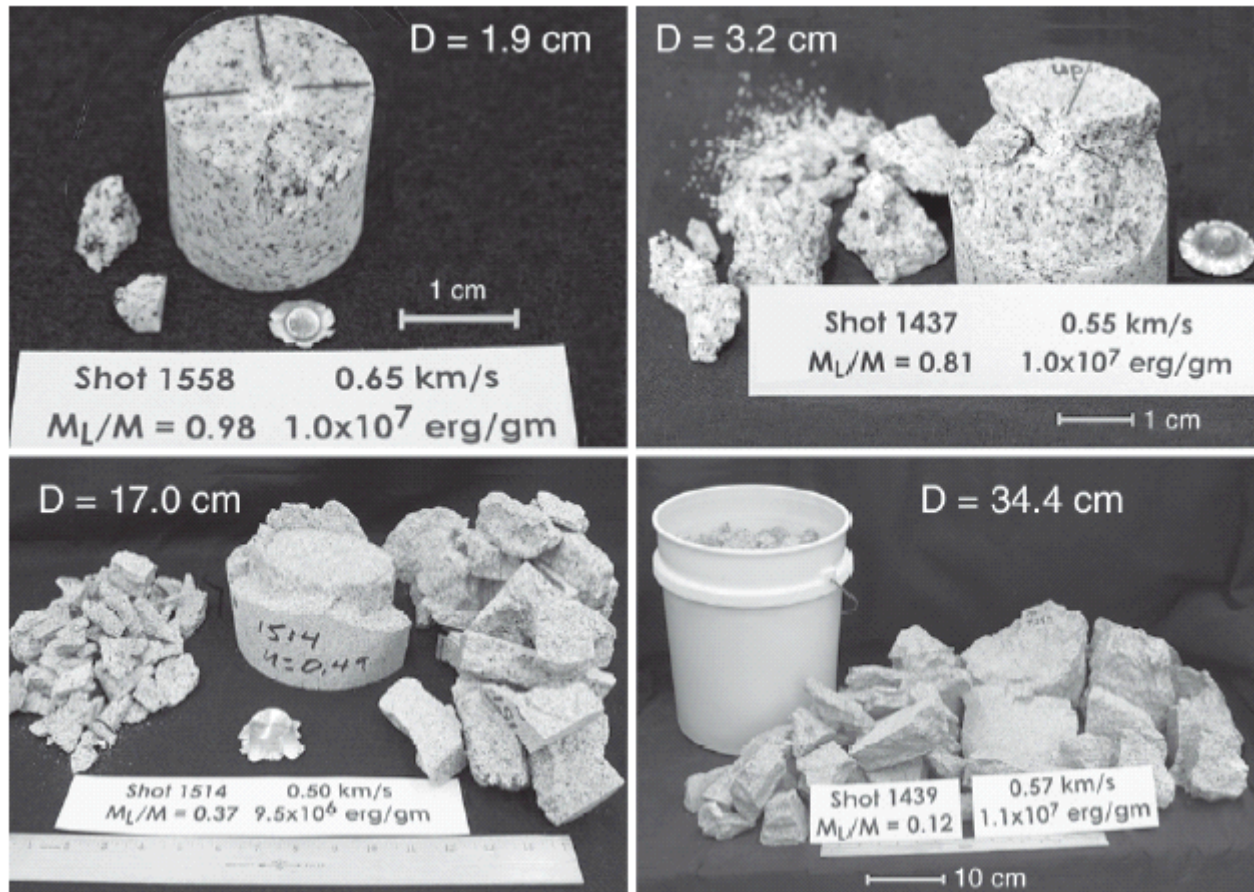
とすると、ランダムに選んだ体積 $V$ の中に、応力 $\sigma$ 以下で成長し始めるひびを含む確率は、

$$1 - \exp[-(\sigma / \sigma_{min})^m]$$

ただし、

$$\sigma_{min} = (kV)^{-1/m} \quad \leftarrow \text{強度のサイズ効果}$$

# ターゲットサイズ効果



(Housen and Holsapple, 1999)

標的/弾丸比を固定して速度も(ほぼ)一定で系の大きさだけ変化した衝突実験

大きいものほど壊れやすい、細かく壊れる

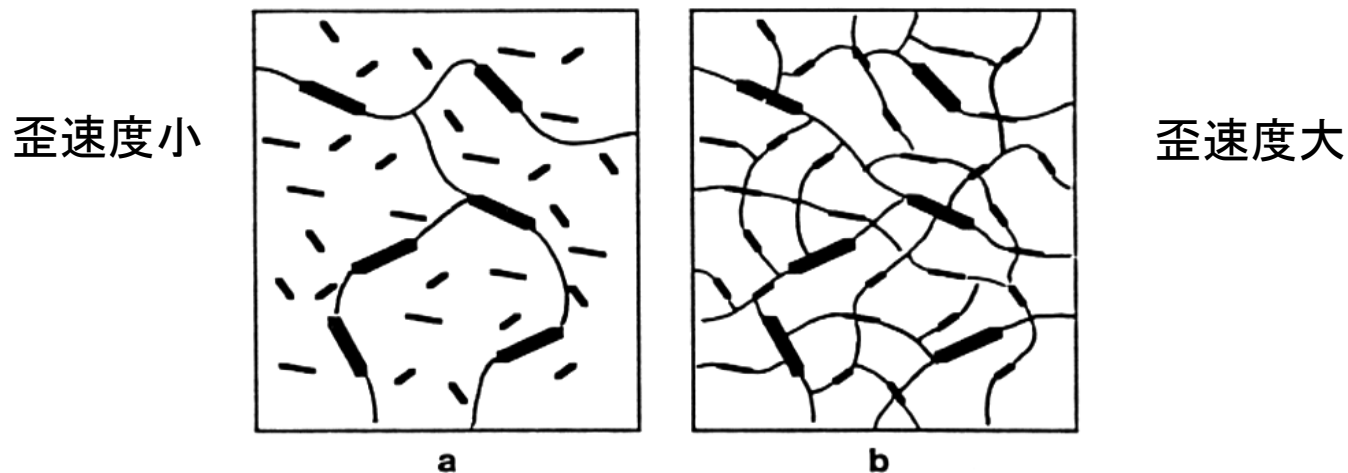
# Grady-Kippの破壊モデル (1980)

- 脆性物質は、内在するひび(のもと)が成長して、引張りで壊れる。

$$n(\sigma) = k \sigma^m$$

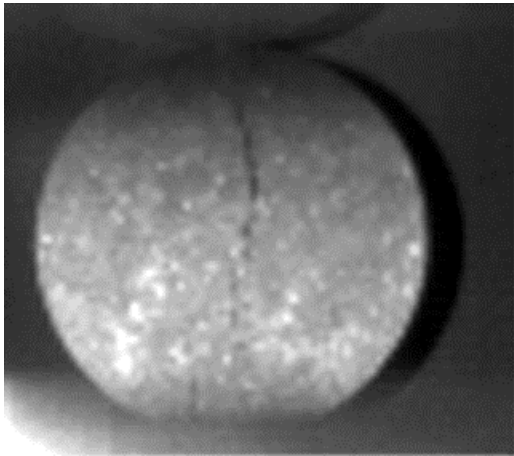
歪 $\varepsilon$ 以下で活性化するひびの個数  $n(\varepsilon)$  は  $n(\varepsilon) = k\varepsilon^m$  とかけられる。

- ひびは、一定速度  $C_g$  で時間とともに成長する。
- ひびの空間密度により定義されるdamage "D",  $\sigma_D = \sigma(1 - D)$

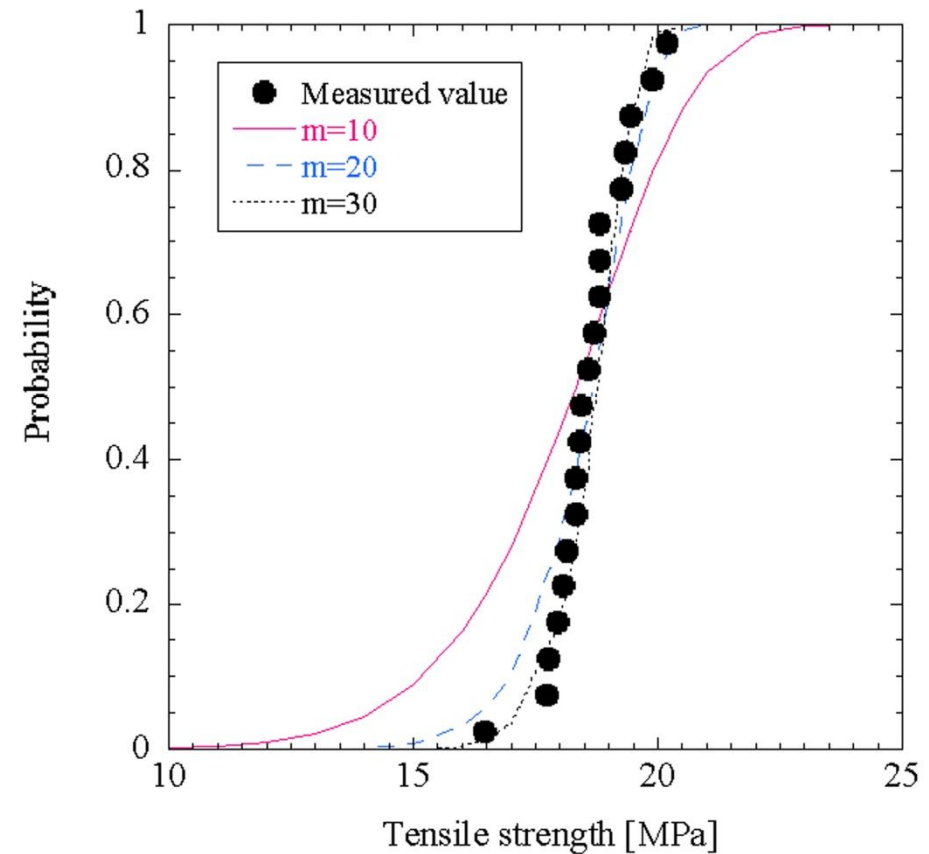


$$\text{破片サイズ} \propto (\text{歪速度})^{-m/(m+3)}$$

# Weibull (1939) の手法による

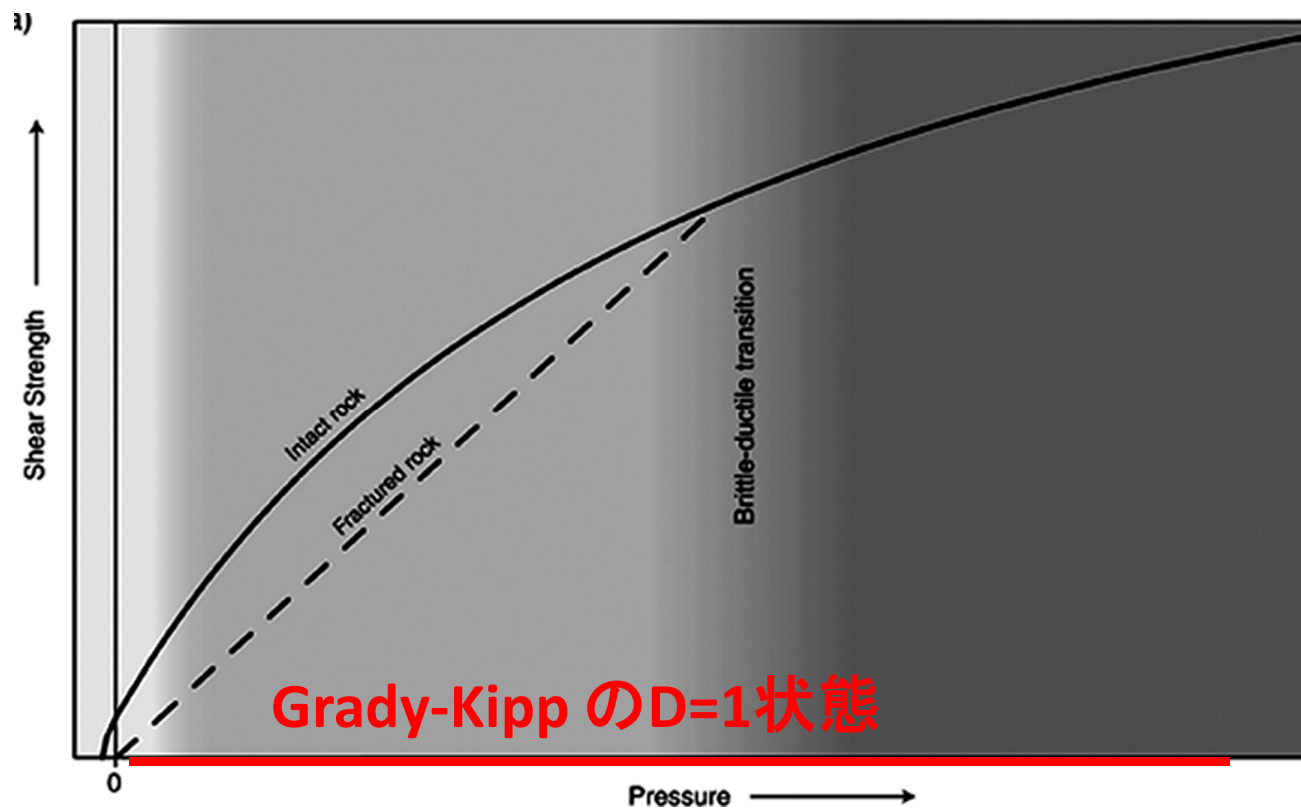


直径10 mm、厚み5 mm の  
円板 20 個の圧裂引張強度測定



$$1 - \exp(-\sigma / \sigma_0)^m$$
$$m = 15 - 17$$

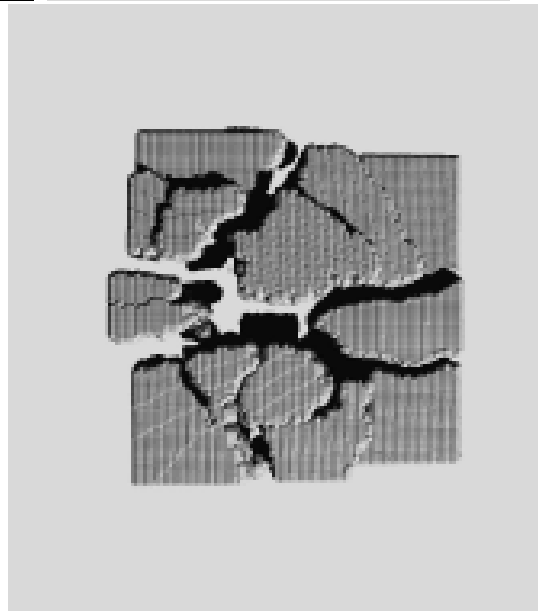
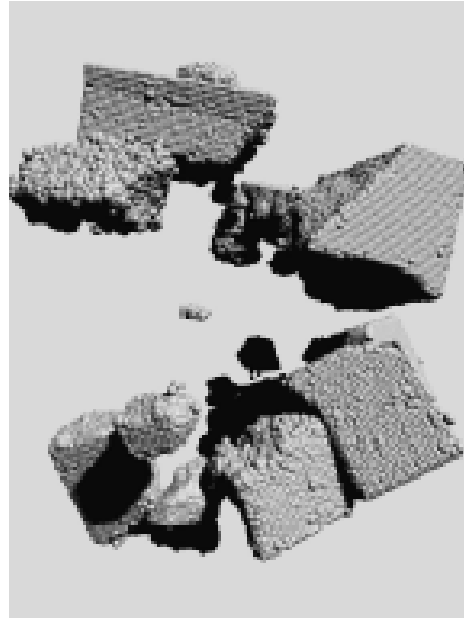
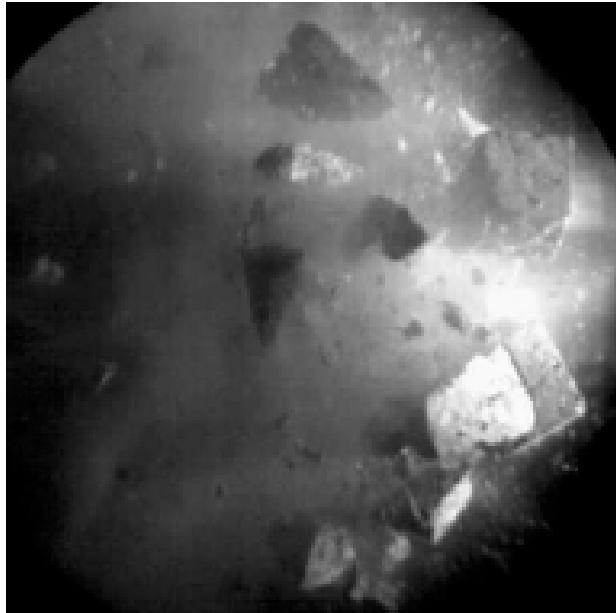
(Nakamura, Michel, and Setoh, 2007)



G-Kモデルを採用した数値計算 破壊された部分が流体として振る舞う

⇒ Shear が重要なクレーター形成過程のシミュレーションには向かない  
(Holsapple)

引張破壊が支配的な Disruption向け



(Jutzi et al., 2009)

# シミュレーションと実験の共同研究への道

1. 出会い・タイミング
2. 少なくとも片方に強い熱意・わくわく感
3. 互いの言語を理解しようとする姿勢

